Cosmología / Cosmología Observacional, lecture 14

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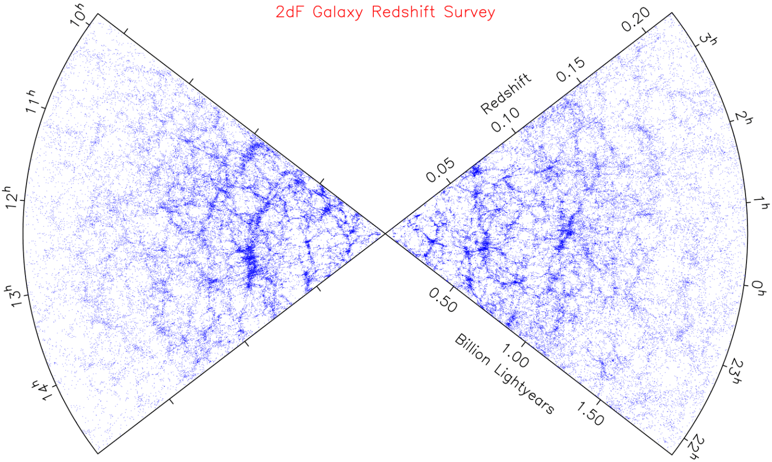
Structures in the Universe (1)

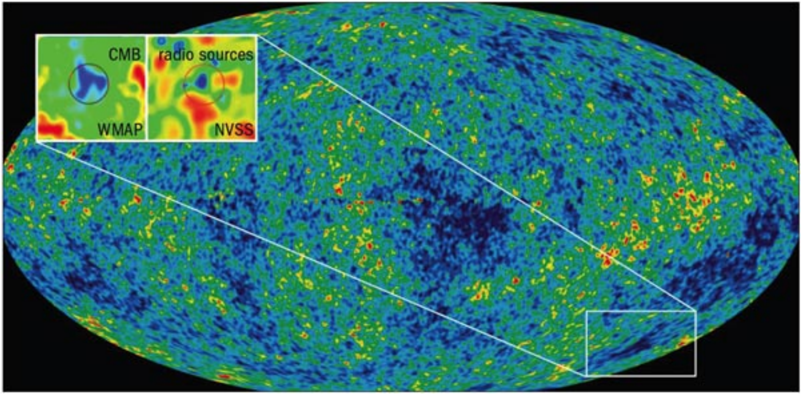
- For the Universe on very large scales, we assumed homogeneity and isotropy.
- The distribution of galaxies on the sky is not uniform or random - they form clusters and groups of galaxies.
- Even clusters of galaxies can be grouped into so-called superclusters.
- The Great Wall is a structure of galaxies with an extent of $\sim 100h^{-1}$ Mpc.
- Galaxy surveys further revealed so-called voids, cosmologically underdense areas with diameters of $\sim 50h^{-1}$ Mpc.
- A larger structure is also the "cold spot" in the CMB, with a radius of 5° and 70 μK colder than the average CMB temperature.

Structures in the Universe (2)



Distribution of galaxies from the 2dF Galaxies Redshift Survey, including the **Great Wall**.

Structures in the Universe (3)



Cold spot in the Cosmic Microwave Background.

Growth of structure (1)

- The temperature fluctuations in the CMB are

$$\frac{\Delta T}{T} \sim 10^{-5} \tag{1}$$

at $z \sim 1000$, requiring small amplitudes for the initial density fluctuations.

- Galaxy clusters in the Universe today show overdensities of more than 200 on scales of $1.5h^{-1}$ Mpc.

- We define the **relative density contrast**

$$\delta(\vec{r}, t) = \frac{\rho(\vec{r}, t) - \bar{\rho}(t)}{\bar{\rho}(t)}, \tag{2}$$

with $\bar{\rho}(t)$ the mean cosmic matter density in the Universe.

Growth of structure (2)

- While the expansion of the Universe is regulated by the mean matter density $\bar{\rho}(t)$, the density fluctuations $\Delta\rho(\vec{r}, t) = \rho(\vec{r}, t) - \bar{\rho}(t)$ generate a local gravitational field regulating the growth of structures.
- In the following, we will consider weak gravitational fields, where the Newtonian approximation applies.
- As the Poisson equation is linear, it can then be separately applied to the background and fluctuating components.
- As a result, density fluctuations grow over time due to self-gravity, while underdense regions decrease their density contrast due to further expansion.

Growth of structure (3)

- Due to local mass conservation, the evolution of matter is governed by the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0. \tag{3}$$

- Local momentum conservation further implies the Euler equation

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{\nabla P}{\rho} - \nabla \Phi. \tag{4}$$

- On large scales, pressure effects can be neglected, i.e. $P = 0$.

- The gravitational field follows from Poisson's equation,

$$\nabla^2 \Phi = 4\pi G \rho. \tag{5}$$

Growth of structure (4)

- An exact solution to this problem is given by our model for the expanding Universe, with

$$\vec{v}(\vec{r}, t) = H(t)\vec{r}, \tag{6}$$

where

$$\vec{r}(t) = a(t)\vec{x} \tag{7}$$

and

$$\rho(t) \propto a^{-3}. \tag{8}$$

- In the general case, the velocity field is given as

$$\vec{v} = \frac{d}{dt}\vec{r} = \dot{a}\vec{x} + a(t)\dot{\vec{x}} = \frac{\dot{a}}{a}\vec{r} + \vec{u}(\vec{r}, t), \tag{9}$$

where $\vec{u}(\vec{r}, t)$ denotes the **peculiar velocity**.

Growth of structure (5)

- We now aim to transform these equations into co-moving coordinates.
- With $\vec{r} = a\vec{x}$, we recall that

$$d\vec{r} = \dot{a}\,dt\,\vec{x} + a\,\dot{\vec{x}}dt = a\,H\,\vec{x}\,dt + a\,d\vec{x}. \tag{10}$$

- With $\nabla_r = \frac{1}{a}\nabla_x$, we can show for the differential of an arbitrary function f:

$$df = \frac{\partial f}{\partial t}dt + \nabla_r f \cdot d\vec{r} = \frac{\partial f}{\partial t}dt + \nabla_r f \cdot a\left(H\vec{x}dt + d\vec{x}\right) \tag{11}$$

$$= \left(\frac{\partial f}{\partial t} + H\vec{x}\cdot\nabla_x f\right)dt + \nabla_x f \cdot d\vec{x}. \tag{12}$$

- The latter implies the transformation

$$\left(\frac{\partial}{\partial t}\right) f(\vec{r}, t) \to \left(\frac{\partial}{\partial t}\right) f(a\vec{x}, t) + \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x f(a\vec{x}, t). \tag{13}$$

Growth of structure (6)

- Employing these transformations, the continuity equation reads

$$\frac{\partial \rho}{\partial t} + \frac{3\dot{a}}{a}\rho + \frac{1}{a}\nabla \cdot (\rho \vec{u}) = 0, \tag{14}$$

where from now all spatial derivatives are with respect to $\vec{x}$.

- Employing $\rho = \bar{\rho}(1+\delta)$ and $\bar{\rho} \propto a^{-3}$, one can show

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla \cdot [(1+\delta)\vec{u}] = 0. \tag{15}$$

- The gravitational potential is written as

$$\Phi(\vec{r}, t) = \frac{2\pi}{3} G\bar{\rho}(t)|\vec{r}|^2 + \phi(\vec{x}, t), \tag{16}$$

where the first term describes the gravitational potential in a homogeneous density field for a sphere with radius $|\vec{r}|$, and ϕ its fluctuation.

Growth of structure (7)

- The Poisson equation for the fluctuating field then follows as

$$\nabla^2\phi(\vec{x},t) = 4\pi G a^2(t)\bar{\rho}(t)\delta(\vec{x},t) = \frac{3H_0^2\Omega_m}{2a(t)}\delta(\vec{x},t), \qquad (17)$$

using $\bar{\rho} \propto a^{-3}$ and the definition of Ω_m.

- From the Euler equation taking first order terms, one can show that

$$\frac{\partial\vec{u}}{\partial t} + \frac{(\vec{u}\cdot\nabla)\vec{u}}{a} + \frac{2\dot{a}}{a}(a\vec{x}\cdot\nabla)\vec{u} = -\frac{1}{\bar{\rho}a}\nabla P - \frac{1}{a}\nabla\Phi. \qquad (18)$$

Linear theory (1)

- In the following, we will consider small perturbations $\delta \ll 1$.
- Under this assumption, one can linearize the continuity, Euler and Poisson equation.
- After the linearization, the equations can be combined to eliminate the peculiar velocity $\vec{u}$ and the gravitational potential ϕ.
- We obtain:

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{2\dot{a}}{a}\frac{\partial \delta}{\partial t} = 4\pi G\bar{\rho}\delta. \tag{19}$$

- We note that no spatial derivatives appear in this equation. In linear theory, perturbations grow independently at every place at a fixed rate!

Linear theory (2)

- The solutions of (19) must be of the form

$$\delta(\vec{x}, t) = D(t)\bar{\delta}(\vec{x}). \quad (20)$$

- Here, $\bar{\delta}(\vec{x})$ is an arbitrary function of the spatial coordinates, while $D(t)$ satisfies

$$\ddot{D} + \frac{2\dot{a}}{a}\dot{D} - 4\pi G\bar{\rho}(t)D = 0. \quad (21)$$

- Eq. (21) has an increasing and a decaying solution.
- In the context of structure formation, we are only interested in the growing mode $D_+(t)$. We then have

$$\delta(\vec{x}, t) = D_+(t)\delta_0(\vec{x}). \quad (22)$$

Linear theory (3)

- In linear theory, the spatial shape of the density fluctuations is fixed in co-moving coordinates.
- However, their amplitudes grow as dictated by the **growth factor** $D_+(t)$.
- For arbitrary cosmological parameters, one can show that

$$D_+(a) \propto \frac{H(a)}{H_0} \int_0^a \frac{da'}{[\Omega_m/a' + \Omega_\Lambda a'^2 - (\Omega_m + \Omega_\Lambda - 1)]^{3/2}}, \qquad (23)$$

  where the normalization is obtained requiring $D_+(t_0) = 1$.

- With this normalization, $\delta_0(\vec{x})$ would described the density field today if we were still in the linear regime.

Linear theory (4)

- In a so-called Einstein-de Sitter Universe ($\Omega_m = 1$, $\Omega_\Lambda = 0$), Eq. (21) can be solved explicitly.
- We have $a(t) = (t/t_0)^{2/3}$, implying that

$$\frac{\dot{a}}{a} = \frac{2}{3t} \tag{24}$$

and

$$\bar{\rho}(t) = a^{-3}\rho_{cr} = \frac{3H_0^2}{8\pi G}\left(\frac{t}{t_0}\right)^{-2}. \tag{25}$$

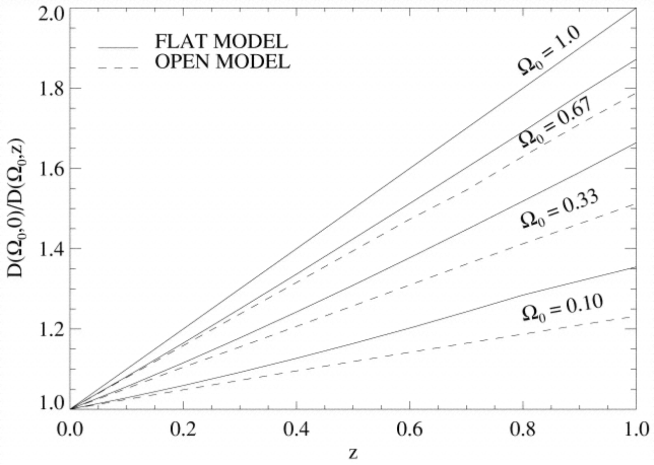
- Using $H_0 t_0 = 2/3$, the equation for the growth factor becomes

$$\ddot{D} + \frac{4}{3t}\dot{D} - \frac{2}{3t^2}D = 0. \tag{26}$$

Linear theory (5)

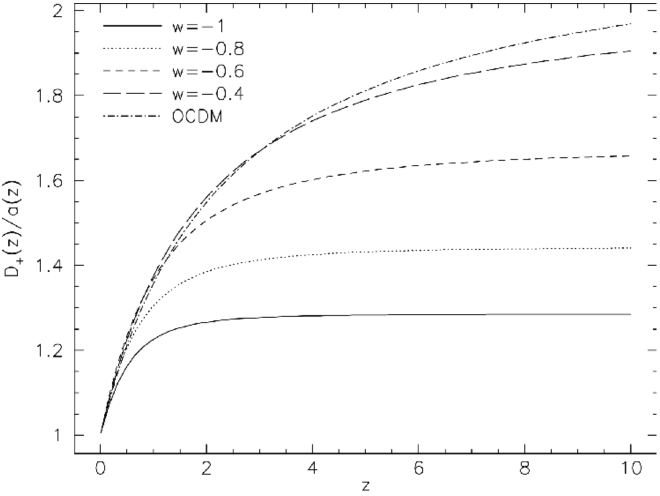
- Eq. (26) is solved using a power-law ansatz
$$D \propto t^q. \tag{27}$$
- We obtain
$$q(q-1) + \frac{4}{3}q - \frac{2}{3} = 0. \tag{28}$$
- The solutions are given as $q = 2/3$ and $q = -1$. The second solution corresponds to decaying solutions and is not relevant here.
- For the increasing solution, we thus have
$$D_+(t) = \left(\frac{t}{t_0}\right)^{2/3} = a(t). \tag{29}$$
- We note that the qualitative behavior is very similar even for cosmologies with $\Omega_\Lambda \neq 0$.

Linear theory (6)



The growth factor for different cosmologies.

Linear theory (7)

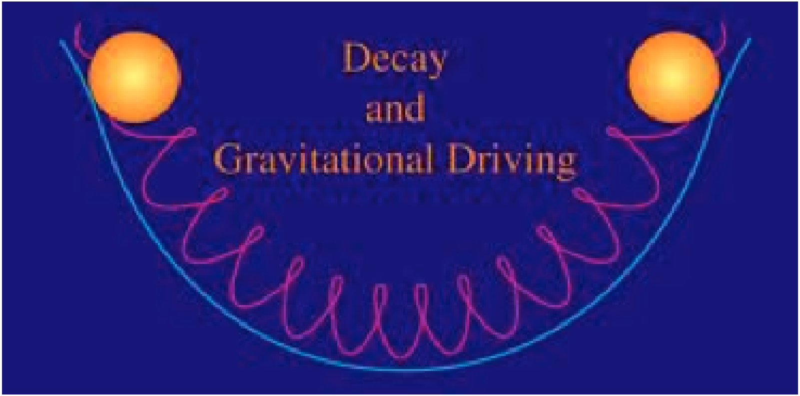


The growth factor (divided by a) in quintessence models with $p = \omega \rho c^2$.

Comparison with observations (1)

- On cluster scales (~ 2 Mpc), the Universe appears non-linear today, with $\delta \gg 1$.
- On scales of superclusters (~ 10 Mpc), we have $\delta \sim 1$.
- As $D_+(a) \propto a$, we thus expect fluctuations of $\delta > 10^{-3}$ at $z = 1000$.
- Then, we should also expect CMB fluctuation of comparable magnitude, $\Delta T/T > 10^{-3}$, while observations show $\Delta T/T \sim 10^{-5}$.
- The CMB observations however reflect only the perturbations of the baryons. Dark matter is expected to have larger perturbations, and the baryons may oscillate in the dark matter potentials.

Comparison with observations (2)



Baryon fluctuations in the dark matter potential due to gravity and photon pressure.