Cosmologia / Cosmologia Observacional, lecture 6

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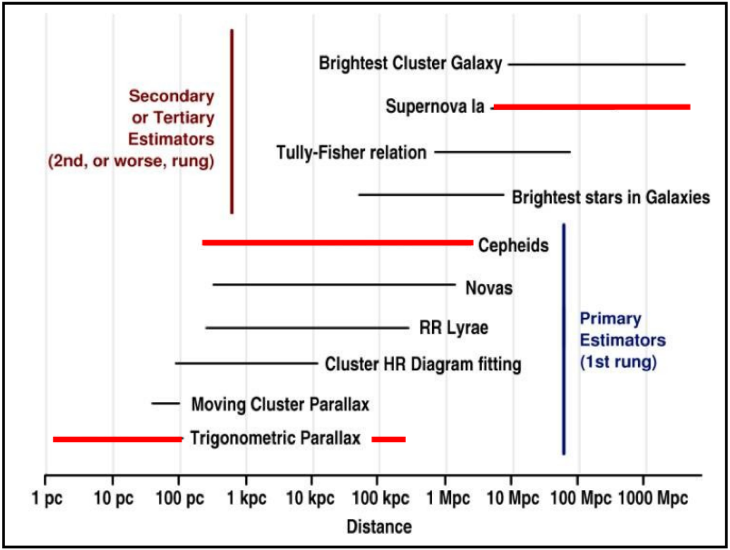
The cosmological distance ladder



The Tully-Fisher relation (1)

- With 21 cm observations of spiral galaxies, **R. Brent Tully and J. Richard Fisher** found in 1977 that the maximum rotation velocity of spirals is closely related to their luminosity as

$$L \propto v_{max}^{\alpha} \tag{1}$$

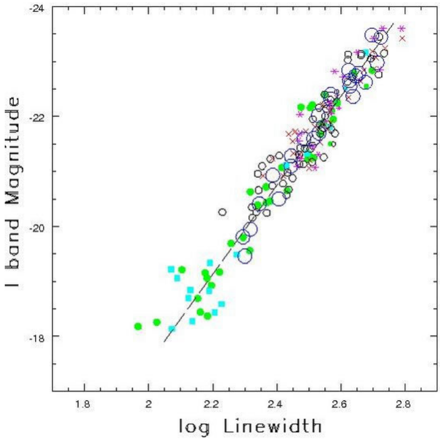
  with $\alpha \sim 4$.

- The larger the wavelength of the used filter, the smaller the dispersion of the relation (less absorption via dust grains at larger wavelengths).

- From this correlation, the luminosity of the spirals can be estimated quite precisely by measuring the rotational velocity.

- The Tully-Fisher relation thus provides a potential **distance measure**.

The Tully-Fisher relation (2)



The Tully-Fisher relation of spiral galaxies.

The Tully-Fisher relation (3)

- To explain the Tully-Fisher relation, we recall that the shapes of the rotation curves in spiral galaxies are very similar.

- In particular, flat rotation curves imply the relation

$$M = \frac{v_{max}^2 r}{G}. \tag{2}$$

- We rewrite this relation as

$$L = \left(\frac{M}{L}\right)^{-1} \frac{v_{max}^2 r}{G}. \tag{3}$$

- Expressing r in terms of the mean surface brightness $\langle I \rangle = L/r^2$, we have

$$L = \left(\frac{M}{L}\right)^{-2} \left(\frac{1}{G^2 \langle I \rangle}\right) v_{max}^4. \tag{4}$$

The Tully-Fisher relation (4)

- This is indeed the Tully-Fisher relation if M/L and $\langle I \rangle$ are the same for all spiral galaxies.
- The second requirement corresponds to **Freeman's law**.
- The first requirement, on the other hand, is at least plausible, as the dark matter profiles are very similar, the ratio of luminous to dark matter may always evolve in a similar fashion.
- While the above derivation is not rigorous, it is nevertheless plausible that a relation like Tully-Fisher should exist.

The Tully-Fisher relation (5)

- For a more **accurate measurement of M/L**, we need to define the radius in which we aim to measure this quantity.
- With R_{25}, we denote the radius at which the surface brightness attains 25 mag/arcsec^2 in the B-band.
- Spirals indeed follow the relation

$$\log\left(\frac{R_{25}}{kpc}\right) = -0.249 M_B - 4.00, \quad (5)$$

independent of the Hubble type.

- Within R_{25}, one can now **measure M/L**, finding 6.2 for Sa's, 4.5 for Sb's and 2.6 for Sc's.
- This is consistent with the presence of bluer, more massive stars in late type spirals, which are more luminous.

The Tully-Fisher relation (6)

- Our derivation of the Tully-Fisher relation assumed constant M/L, with M the total mass.
- Let us assume that the ratio of baryons to dark matter is constant, and that the ratio of stellar mass to luminosity is also similar.
- Then, the Tully-Fisher relation is valid if the **gas mass** can be neglected compared to the stellar mass.
- However, low-mass spirals contain a significant amount of gas, and deviations from Tully-Fisher were found for $v_{max} < 100$ km/s.
- As the luminosity is proportional to the stellar mass M_*, Tully-Fisher implies a relation between v_{max} and M_*.

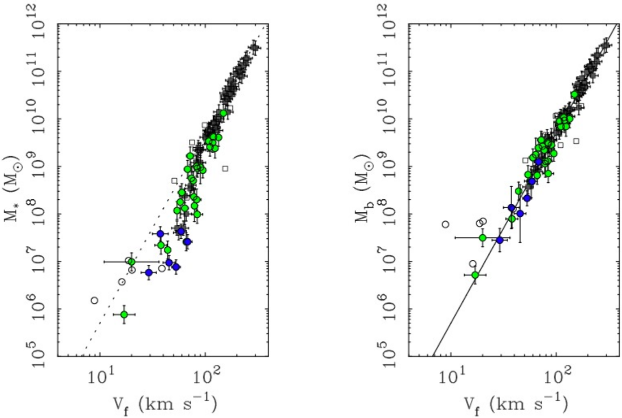
The Tully-Fisher relation (7)

- If the gas mass is obtained from 21 cm observations, one obtains a much tighter correlation

$$M_{disk} = 2 \times 10^9 h^{-2} M_\odot \left(\frac{v_{max}}{100 \text{ km/s}} \right)^4 . \quad (6)$$

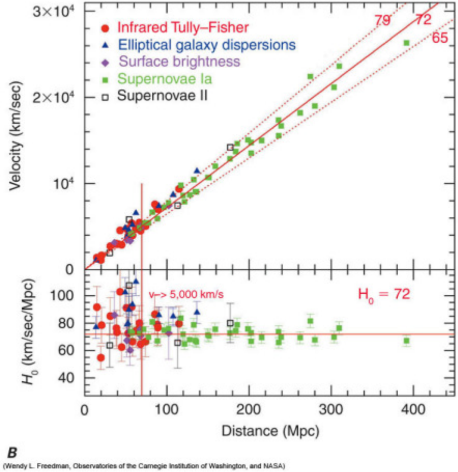
- This relation is referred to as the **baryonic Tully-Fisher relation**.

The Tully-Fisher relation (8)



The baryonic Tully-Fisher relation of spiral galaxies.

Expansion measurements with the Tully-Fisher relation (1)



A Hubble diagram from the Tully-Fisher relation.

Expansion measurements with the Tully-Fisher relation (2)

- While Tully-Fisher allows to extend distance measurements to about 100 Mpc, it is still relatively local.
- The Tully-Fisher measurements thus help with the measurement of H_0 (providing an average over larger scales), but not with the measurement of q_0.

The Faber-Jackson relation (1)

- An analogous relation was found by **Sandra Faber and Roger Jackson** for elliptical galaxies.
- The velocity dispersion in the center of ellipticals, σ_0, scales with luminosity L as

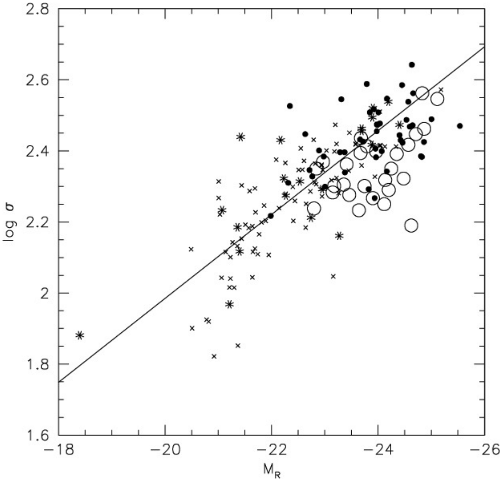
$$L \propto \sigma_0^4 \quad (7)$$

or

$$\log(\sigma_0) = -0.1 M_B + \text{const.} \quad (8)$$

- This relation can be "derived" by similar arguments as the Tully-Fisher relation for spirals. The scatter in this relation is however much larger.

The Faber-Jackson relation (2)



The Faber-Jackson relation of elliptical galaxies.

The fundamental plane (1)

- While the Faber-Jackson relation shows a significant amount of scatter, addition correlations for elliptical parameters were found early on.
- One may thus wonder if a **more fundamental correlation** can be obtained which is scatter-free.
- Indeed, the surface brightness and the effective radius are related as

$$r_e \propto \langle I \rangle_e^{-0.83}, \tag{9}$$

with $\langle I \rangle_e$ the average surface brightness within r_e.

- We then have

$$L = 2\pi r_e^2 \langle I \rangle_e. \tag{10}$$

The fundamental plane (2)

- From this, a relation between L and $\langle I \rangle_e$ follows as

$$L \propto r_e^2 \langle I \rangle_e \propto \langle I \rangle_e^{-0.66} \tag{11}$$

or

$$\langle I \rangle_e \propto L^{-1.5}. \tag{12}$$

- Due to the Faber-Jackson relation, L is related to σ_0, so σ_0, $\langle I \rangle_e$ and r_e are related to each other.
- In this parameter space, elliptical galaxies lie close to a plane defined as

$$r_e \propto \sigma_0^{1.4} \langle I \rangle_e^{-0.85}. \tag{13}$$

The fundamental plane (3)

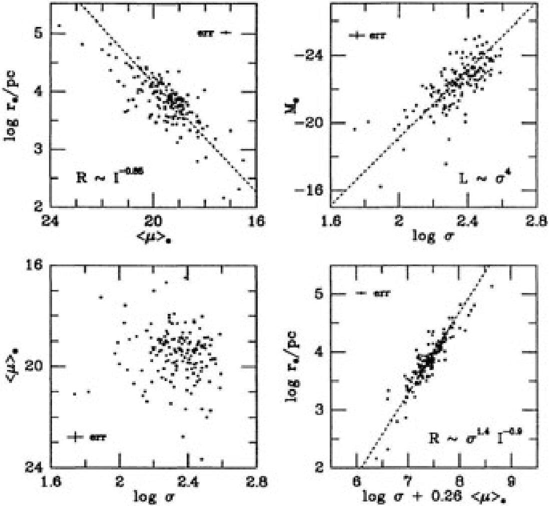
- In logarithmic form, we have

$$\log(r_e) = 0.34\langle\mu\rangle_e + 1.4\log\sigma_0 + \mathrm{const}, \qquad (14)$$

with $\langle\mu\rangle_e$ the average surface brightness within r_e.

- This equation defines the **fundamental plane of elliptical galaxies**.

The fundamental plane (4)



The fundamental plane of elliptical galaxies.

The fundamental plane (5)

- To explain this relation, the mass within r_e can be derived from the virial theorem, yielding $M \propto \sigma_0^2 r_e$.

- Together with

$$L = 2\pi r_e^2 \langle I \rangle_e, \tag{15}$$

we obtain

$$r_e \propto \frac{L}{M} \frac{\sigma_0^2}{\langle I \rangle_e}. \tag{16}$$

- This is consistent with the fundamental plane if

$$\frac{L}{M} \frac{\sigma_0^2}{\langle I \rangle_e} \propto \frac{\sigma_0^{1.4}}{\langle I \rangle_e^{0.85}} \tag{17}$$

or

$$\frac{M}{L} \propto \frac{\sigma_0^{0.6}}{\langle I \rangle_e^{0.15}} \propto \frac{M^{0.3}}{r_e^{0.3}} \frac{r_e^{0.3}}{L^{0.15}}. \tag{18}$$

The fundamental plane (6)

- The fundamental plane is thus consistent with the virial theorem if

$$\left(\frac{M}{L}\right) \propto M^{0.2} \tag{19}$$

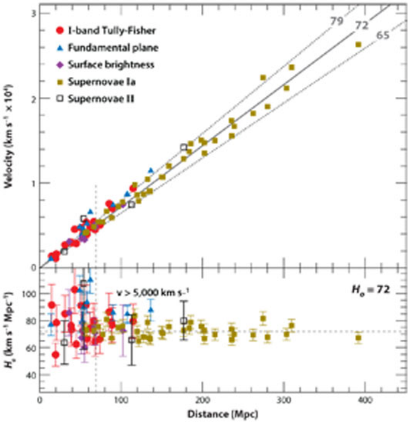
or

$$\left(\frac{M}{L}\right) \propto L^{0.25}, \tag{20}$$

i.e. requiring a slightly increasing mass-to-light ratio with mass.

- Like the Tully-Fisher relation, the fundamental plane is an important tool for distance estimates.

Expansion measurements with the fundamental plane (1)



A Hubble diagram with the Tully-Fisher relation and the fundamental plane.

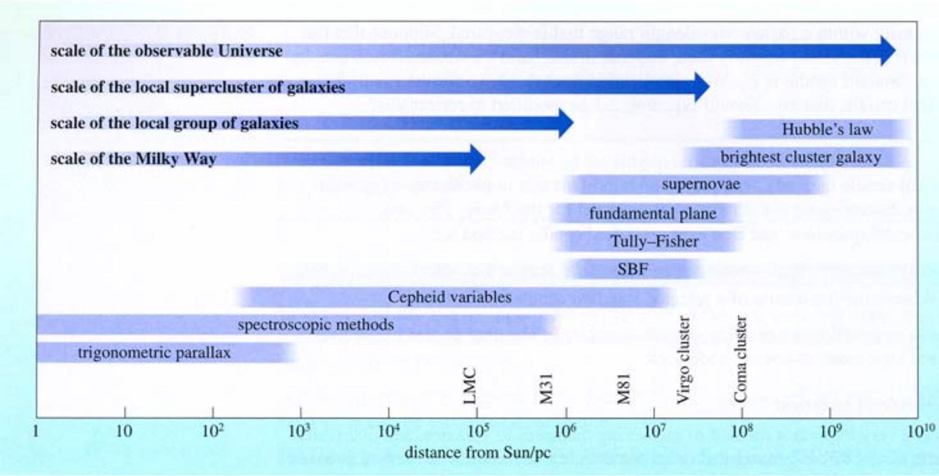
Expansion measurements with the fundamental plane (2)

- Similar to the Tully-Fisher relation, the fundamental plane allows to extend distance measurements to about 100 Mpc.
- Both measurements thus help with the measurement of H_0 (providing an average over larger scales), but not with the measurement of q_0.

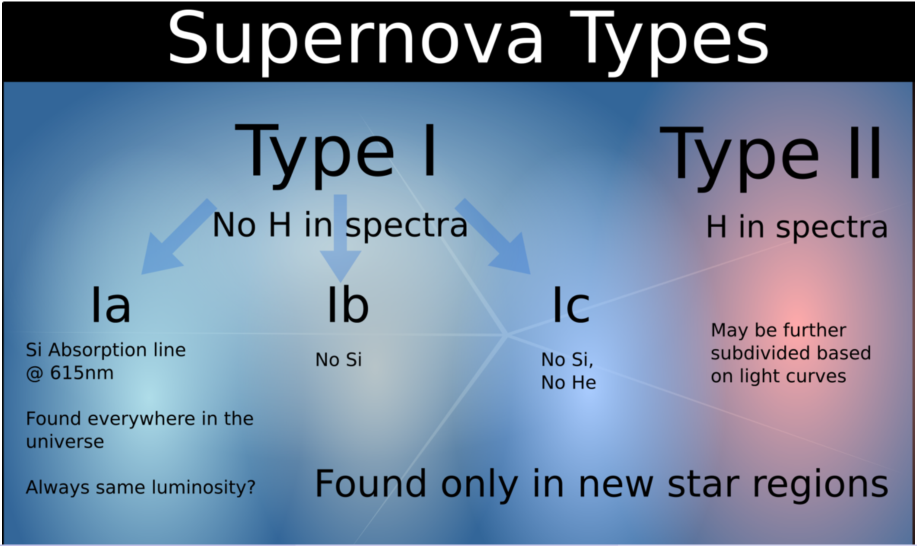
Parameter determination via the luminosity distance

- With Cepheids, we can probe cosmological distances up to ~ 2 Mpc.
- Using the Tully-Fisher relation or the fundamental plane, we can probe distances up to ~ 100 Mpc.
- A distance of 100 Mpc corresponds to a redshift of $z \sim 0.025$, which is very small.
- In both cases, we can thus only probe the current expansion, allowing a measurement of the Hubble constant H_0.
- To determine further cosmological parameters, we need to measure the evolution of the expansion, and measure the luminosity distance at higher redshifts!

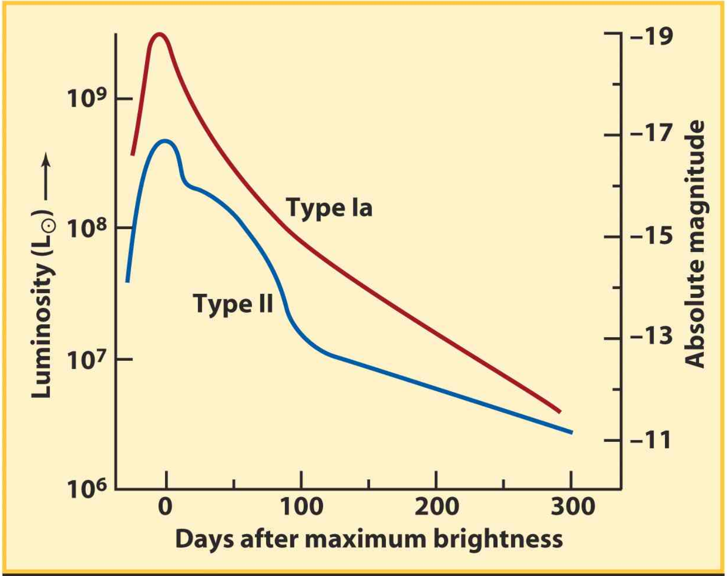
The cosmological distance ladder



Supernova types

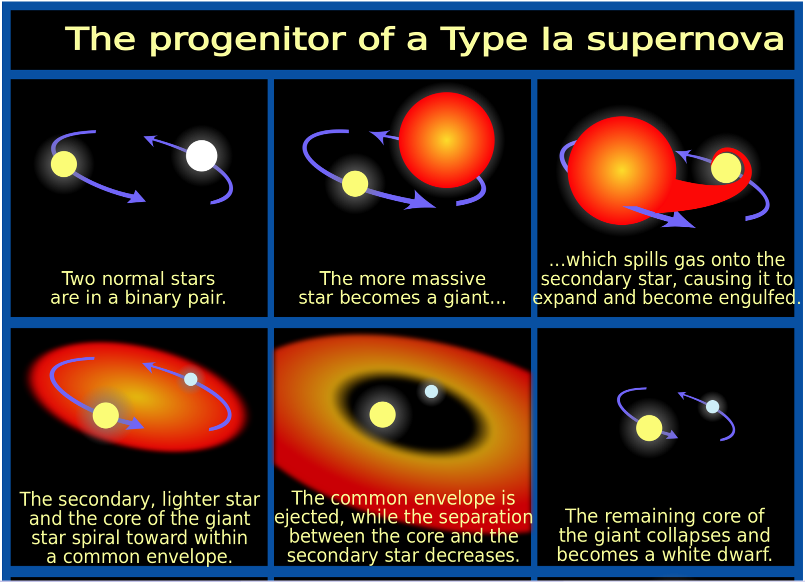


Supernova lightcurves

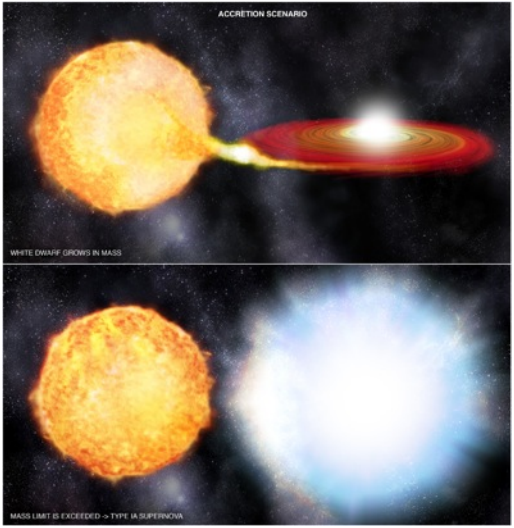


Type Ia supernova in NGC 4526

Origin of type Ia supernovae



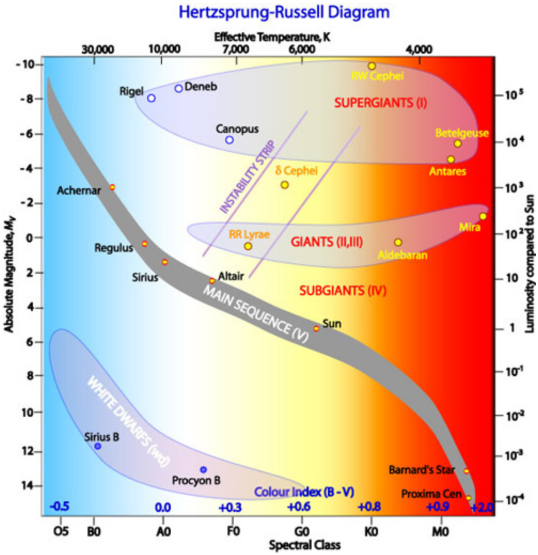
Type Ia supernova explosion (1)



Type Ia supernova explosion (2)

- The type Ia supernova explosion results from the accretion of matter onto a white dwarf.
- When the white dwarf reaches a critical mass, it is no longer stable. The material collapses and an explosion occurs.
- As the occasion always occurs at the same critical mass, it always has the same energy.

White Dwarfs in the Herzsprung-Russell diagram



Properties of White Dwarfs

- From Stefan-Boltzmann's law:

$$L = 4\pi\sigma R^2 T^4. \tag{21}$$

- At the same temperature, different luminosities imply different radii.
- White Dwarfs: $R \sim 0.008 - 0.02\ R_\odot$
- Typical masses: $0.17 - 1.33\ M_\odot$
- Typical densities: $\sim 10^6$ g/cm^3
- Distances between the nuclei: $\sim 10^{-3}$ nm (size hydrogen atoms: $R_B \sim 0.05$ nm)

Heisenberg´s Uncertainty Relation

- Quantum-mechanical particles obey the Uncertainty Relation:

$$\Delta x \Delta p \sim \hbar \quad (22)$$

- $\hbar = 1.05 \times 10^{-34}$ Js is the reduced Planck constant.

Implications for White Dwarfs (1)

- Uncertainty relation:

$$\Delta p \Delta x \sim \hbar \tag{23}$$

- Mean particle distance in a White Dwarf:

$$\Delta x \sim n^{-1/3}, \qquad n \sim \frac{N}{R^3} \tag{24}$$

- Electron kinetic energy resulting from Uncertainty Principle:

$$E_{kin} \sim N\frac{\Delta p^2}{2m_e} \sim \frac{N\hbar^2 n^{2/3}}{2m_e} \sim \frac{\hbar^2 N^{5/3}}{2m_e R^2} \sim \frac{\hbar^2 M^{5/3}}{2m_e R^2 m_p^{5/3}} \tag{25}$$

with $N \sim M/m_p$.

Implications for White Dwarfs (2)

- Potential energy:

$$E_{pot} \sim -\frac{GM^2}{R} \tag{26}$$

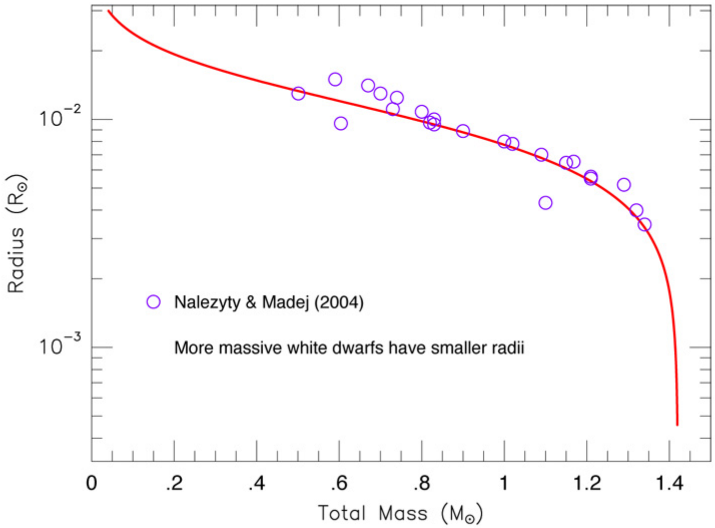
- Potential energy and electron kinetic energy should be approximately equal:

$$E_{pot} \sim -\frac{GM^2}{R} \sim E_{kin} \sim \frac{\hbar^2 M^{5/3}}{2m_e R^2 m_p^{5/3}} \tag{27}$$

- We obtain the mass-radius relationship for White Dwarfs:

$$R \propto M^{-1/3} \tag{28}$$

The mass-radius relation for White Dwarfs



Limit of validity (1)

- The mass-radius relation implies a decreasing radius with increasing mass.
- The escape velocity on the surface of the White Dwarf is given as:

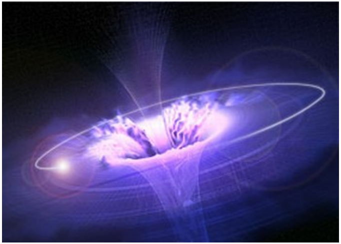
$$v_{esc} = \sqrt{\frac{2GM}{R}} \propto M^{2/3} \tag{29}$$

- From Special Relativity, we know that no physical velocity can be larger than the speed of light c.
- From the condition $v_{esc} = c$, we obtain the radius for which no light can escape from the White Dwarf:

$$R = \frac{2GM}{c^2} \tag{30}$$

- The radius derived above is the Schwarzschild radius for spherically symmetric black holes.

Limit of validity (2)



- No matter or light can escape from within the Schwarzschild radius.
- For stars with the mass of the sun, the Schwarzschild radius is given as $R_S = 3$ km.
- The existence of a critical radius shows that a transition takes places and White Dwarfs no longer are stable when masses are too high!
- Original derivation: John Michell (1785), Pierre Simon de Laplace (1795) (before Einstein)