Cosmologia / Cosmologia Observacional, lecture 7

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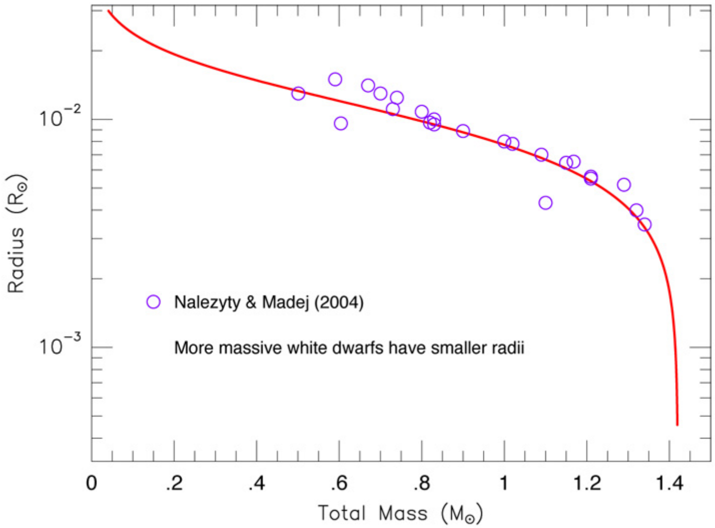
The mass-radius relation for White Dwarfs



Understanding the mass limit (1)

- When approaching the mass limit, the velocities in the White Dwarf become higher and start approaching the speed of light c.
- We therefore need a relativistic treatment to understand this regime.
- From the Uncertainty Principle, our expression for the momentum is given as

$$\Delta p = \frac{\hbar}{n^{-1/3}} \sim \frac{\hbar}{(N/R^3)^{-1/3}}. \tag{1}$$

- In special relativity, the kinetic energy of one particle follows as $E = \Delta p\, c$.
- The kinetic energy of all particles then follows as

$$E_{kin} = N \Delta p\, c = \frac{N\hbar c}{(N/R^3)^{-1/3}}. \tag{2}$$

Understanding the mass limit (2)

- Expressing the number of particles as $N = M/m_p$, we obtain

$$E_{kin} = \frac{M\hbar c}{m_p\left(M/(m_p R^3)\right)^{-1/3}}. \tag{3}$$

- The kinetic energy should be approximately equal to the gravitational energy, i.e.

$$\frac{M\hbar c}{m_p\left(M/(m_p R^3)\right)^{-1/3}} \sim \frac{GM^2}{R}. \tag{4}$$

- Solving for M yields

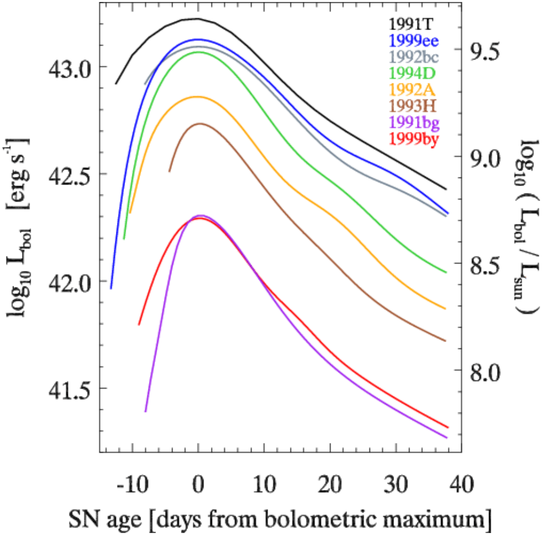
$$M \sim \left(\frac{\hbar c}{G}\right)^{3/2} m_p^{-2} \sim 1.7\, M_\odot. \tag{5}$$

- A more accurate calculation yields 1.39 $M_\odot$.

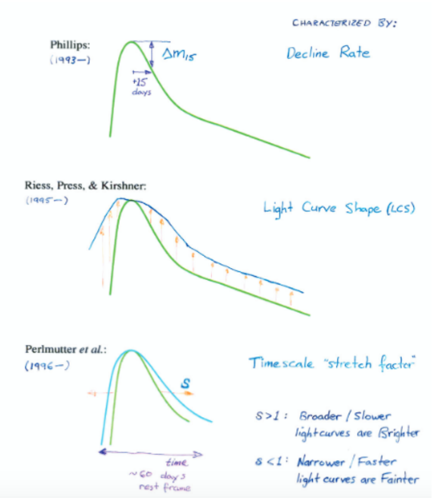
Understanding the mass limit (3)

- The derivation shows that White Dwarfs can only exist up to a critical mass, the Chandrasekhar mass limit.
- When this mass is exceeded by accretion, the White Dwarf is no longer stable, but starts collapsing as a result of gravity.
- At high enough densities, the Uncertainty Principle becomes relevant for the protons and neutrons. The neutron pressure then stabilizes the star, collapse stops.
- When the collapse stops, the kinetic energy from the collapse is released as thermal energy, the White Dwarf explodes!
- This happens always with (roughly) the same mass, and therefore a similar energy for the explosion.

Type Ia supernova lightcurves

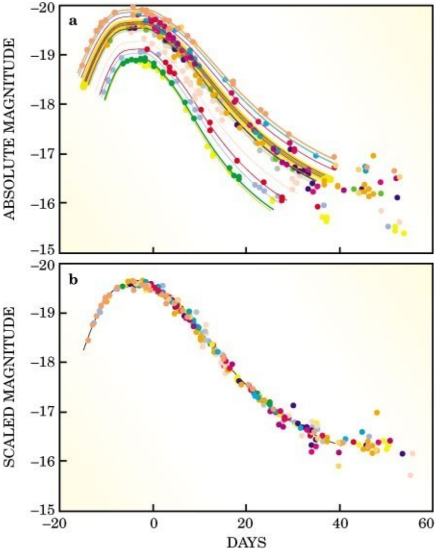


Unterstanding the light curves

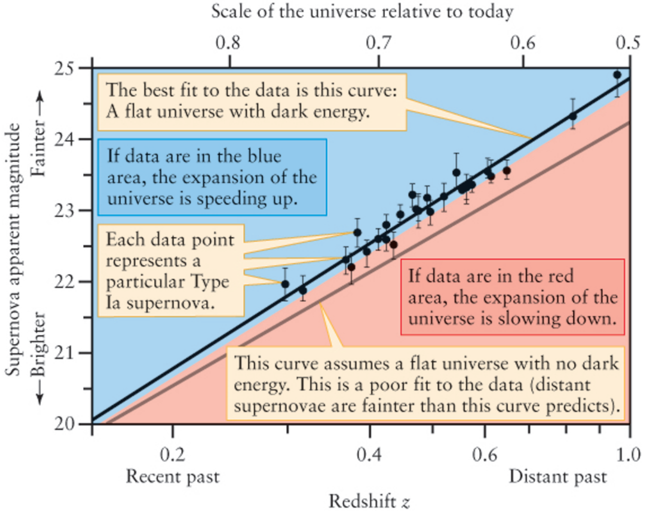


From the Nobel lecture by Saul Permutter (see online material).

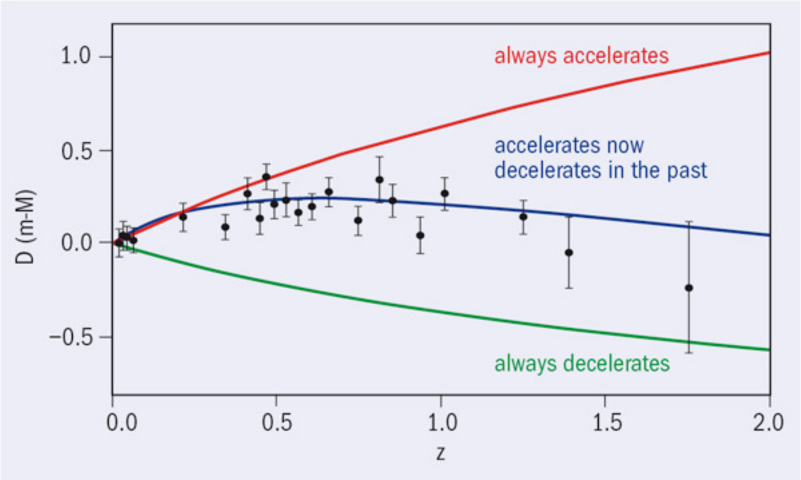
Standardizing the light curves → standard candles



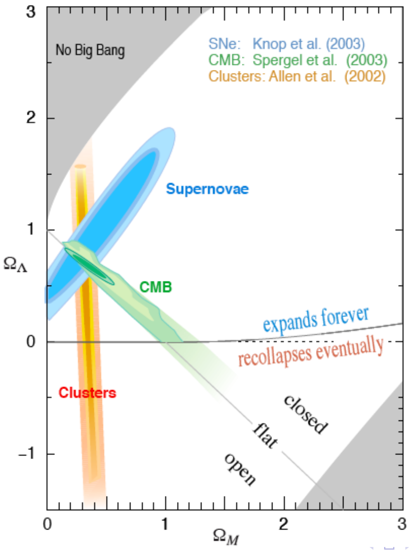
A Hubble diagram with type Ia supernovae



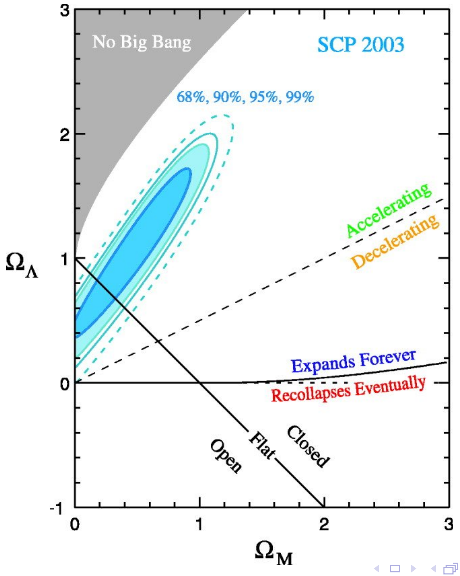
Acceleration vs deceleration



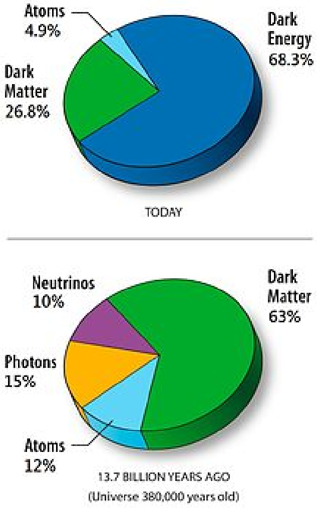
Implications of the supernova measurements (1)



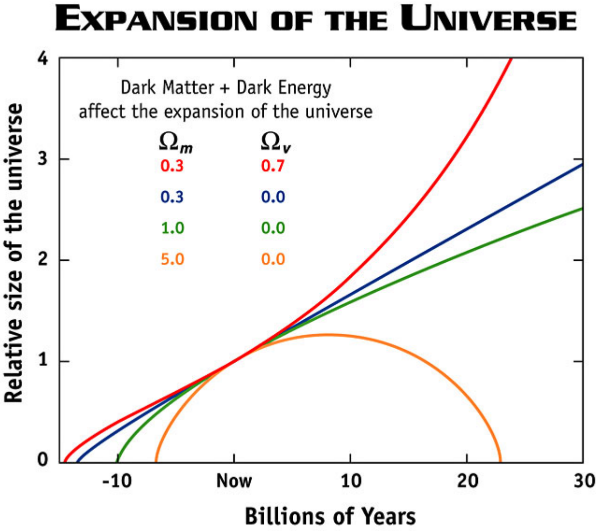
Implications of the supernova measurements (2)



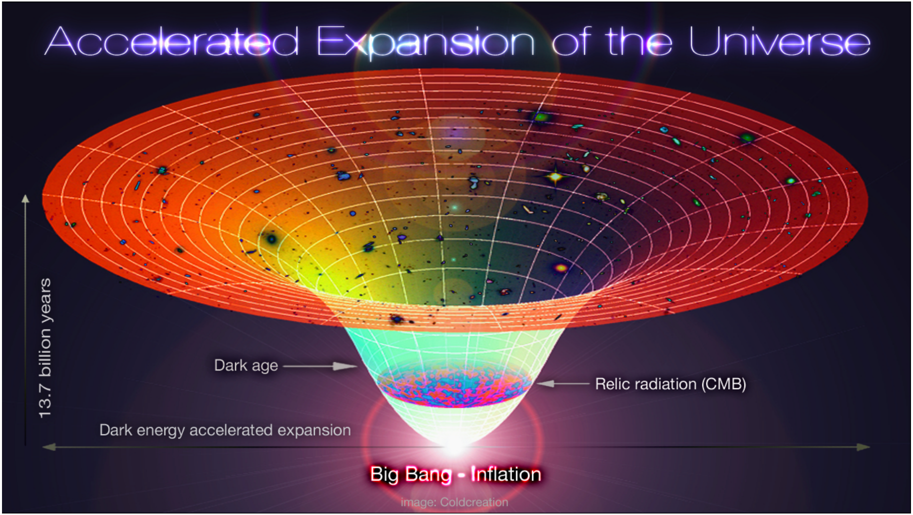
Implications of the supernova measurements (3)



Implications of the supernova measurements (4)



Implications of the supernova measurements (5)



Implications of the supernova measurements (6)

- With the type Ia supernovae, we can probe cosmic expansion beyond redshift 1, to distances larger than ~ 7000 Mpc.
- From the measurements, we can obtain the deceleration parameter, and find that $q_0 < 0$.
- To break the degeneracies in the parameters Ω_Λ and Ω_m, we need additional datasets, in particular from the CMB and from galaxy clusters.

Implications of the supernova measurements (7)

Photo: Ariel Zambelich, Copyright © Nobel Media AB

Saul Perlmutter

Photo: Belinda Pratten, Australian National University

Brian P. Schmidt

Photo: Homewood Photography

Adam G. Riess

Physics nobel prize 2011 for the discovery of the accelerated expansion.

Implications of the supernova measurements (8)

Mario Hamuy: Premio Nacional de las Ciencias Exactas 2015.

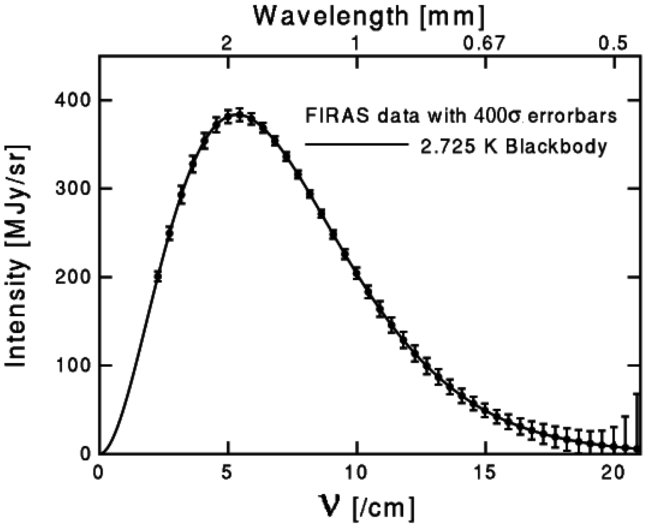
Complementary measurements

- As we have seen, the supernova observations tell us that $q_0 < 0$, but it is hard to disentangle Ω_m and Ω_Λ.
- So far, we also haven't determined Ω_{rad}.
- We will therefore need complementary measurements to disentangle these parameters. This will be done using the cosmic microwave background.

The cosmic microwave background (1)

- A large number of cosmological information comes from the observation of the cosmic microwave background (CMB).
- 1948: Prediction of the CMB by Alpher, Herman and Gamov for Hot Big Bang models.
- 1965: Detection of the background by Penzias and Wilson as a radio excess (nobel prize 1978).
- Temperature of the background: $T_{\rm CMB,0} = 2.725 \pm 0.001$ K.
- Most perfect blackbody ever observed or produced in lab!

The cosmic microwave background (2)



The cosmic microwave background (3)

- The energy density of the CMB is given as

$$e_{\mathrm{rad},0} = aT^4_{\mathrm{CMB},0}, \tag{6}$$

with $a = \frac{4\sigma}{c}$ the radiation constant, and σ the Stefan-Boltzmann constant.

- From $T_{\mathrm{CMB},0} = 2.725$ K, we thus obtain

$$e_{\mathrm{rad},0} \sim 4.2 \times 10^{-13} \mathrm{~erg~cm}^{-3}. \tag{7}$$

The cosmic microwave background (4)

- From the critical density

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G}, \tag{8}$$

with $H_0 \sim 70$ km/s/Mpc, we have

$$\Omega_{\rm rad,0} = \frac{e_{\rm rad,0}}{c^2 \rho_{cr,0}} \sim 4.2 \times 10^{-5}. \tag{9}$$

The cosmic microwave background (5)

- CMB spectrum today:

$$\epsilon_0(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{\exp\left(\frac{h\nu}{k_B T_{\mathrm{CMB},0}}\right)-1}. \tag{10}$$

- The photons at frequency ν at scale factor a were diluted by expansion and redshifted. With $a(t_0)=1$, the respective quantities today are thus

$$\nu_0 = \nu a, \qquad d\nu_0 = d\nu a. \tag{11}$$

- Considering dilution and redshifting, the energy density of photons with frequency ν at scale factor $a<1$ was

$$\epsilon_a(\nu)d\nu = a^{-3}a^{-1}\left(\epsilon_0\left(\nu a\right)d\nu a\right) = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{\exp\left(\frac{a\,h\nu}{k_B T_{\mathrm{CMB},0}}\right)-1}. \tag{12}$$

The cosmic microwave background (6)

- Defining

$$T_{\mathrm{CMB}}(a) = \frac{T_{CMB,0}}{a}, \tag{13}$$

we thus have

$$\epsilon_a(\nu)d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{\exp\left(\frac{h\nu}{k_B T_{\mathrm{CMB}}(a)}\right) - 1}. \tag{14}$$

- At every redshift $z = a^{-1} - 1$, the CMB spectrum can thus be written as a blackbody spectrum.
- From cosmology, we know that $e_{\mathrm{rad}} \propto a^{-4}$. From the Planck law, we expect $e_{\mathrm{rad}} \propto T^4$.
- Both relations are consistent, as $T_{CMB} \propto a^{-1}$.

The photon-to-baryon ratio (1)

- We have just seen that the present energy in the CMB is $e_{\rm rad,0} \sim 4.2 \times 10^{-13}$ erg cm^{-3}.
- The typical energy of a CMB photon is

$$E_{\rm mean} \sim 3k_B T_{\rm CMB,0} \sim 7.0 \times 10^{-4} \text{ eV}. \quad (15)$$

- Considering 1 eV$=1.6 \times 10^{-12}$ erg, the present number density of photons is

$$n_{\gamma,0} \sim \frac{e_{\rm rad,0}}{E_{\rm mean}} \sim 3.7 \times 10^2 \text{ cm}^{-3}. \quad (16)$$

The photon-to-baryon ratio (2)

- The density parameter of the total non-relativistic matter corresponds to $\Omega_{m,0} \sim 0.3$. From spiral galaxies and galaxy clusters, the ratio of baryonic to dark matter corresponds to $\sim 16\%$.

- The density parameter of the baryons is thus

$$\Omega_{B,0} \sim 0.05. \tag{17}$$

- The baryons consist of $\sim 76\%$ hydrogen, 24% helium and a small amount of heavy elements. The mean mass per baryon is thus $\sim 1.2\ m_H$, with m_H the mass of the hydrogen atom.

- We thus estimate the number density of the baryons as

$$n_{B,0} \sim \frac{\Omega_B \rho_{cr,0}}{1.2 m_H} \sim 2.4 \times 10^{-7}\ \mathrm{cm}^{-3}. \tag{18}$$

The photon-to-baryon ratio (3)

- From our estimates, we obtain the photon-to-baryon ratio as

$$\frac{n_{\gamma,0}}{n_{B,0}} \sim 1.5 \times 10^9. \tag{19}$$

- We thus have many more photons than baryons in the Universe.
- Both the number density of the photons and the number density of the baryons evolves as a^{-3}, so this ratio is constant in time!
- We will see that the photon-to-baryon ratio can be independently measured from Big Bang Nucleosynthesis.

Origin of the CMB (1)

- We have seen that the temperature of the CMB scales as a^{-1}, implying that photons were much more energetic in the early Universe.
- The early Universe has consisted of an ionized plasma consisting of ionized nuclei and free electrons, intensely coupled to the photons.
- Assuming a fully ionized plasma, the number density of electrons was

$$n_e \sim n_B a^{-3}. \tag{20}$$

- The mean free path for interactions via Thomson scattering was thus

$$l_{\mathrm{mfp}} \sim \frac{1}{n_e \sigma_T}, \tag{21}$$

with $\sigma_T = 6.65 \times 10^{-25}$ cm^2 the Thomson scattering cross section.

Origin of the CMB (2)

- At a redshift of $z \sim 1000$, assuming a fully ionized plasma, the mean free path of the photons was thus approximately

$$l_{\rm mfp}(z = 1000) \sim \frac{1}{n_e(z = 1000)\sigma_T} \sim 6.3 \times 10^{21} \text{ cm.} \qquad (22)$$

- From $a \propto t^{2/3}$, we can estimate the age of the Universe at that time as

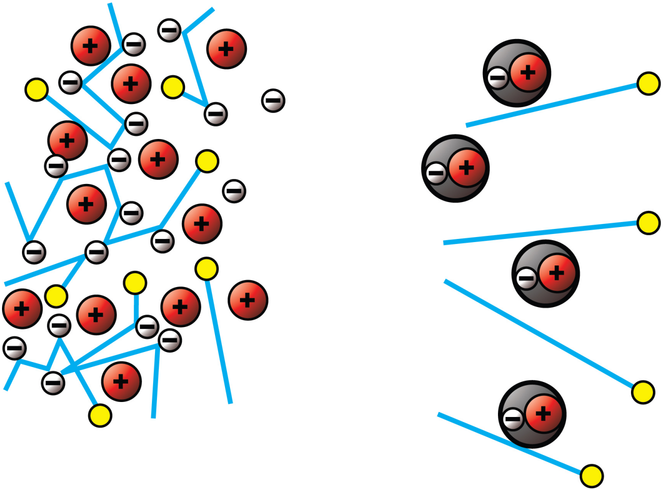
$$t(z = 1000) \sim \left(\frac{1}{1001}\right)^{3/2} t_0 \sim 4.3 \times 10^5 \text{ yrs.} \qquad (23)$$

- In the absence of scattering, the light could have traveled a maximum distance of

$$l_{\rm max} \sim c\, t(z = 1000) \sim 4.1 \times 10^{23} \text{ cm.} \qquad (24)$$

- As $l_{\rm max} \gg l_{\rm mfp}$, the light must have scattered many times!

Origin of the CMB (3)



Scattering in the plasma (left) vs neutral gas (right).

Origin of the CMB (4)

- When the protons and electrons in the Universe recombined, Thomson scattering stopped, and the photons could travel freely throughout the Universe.
- To understand the origin of the CMB, we must therefore understand how the Universe has turned into a neutral state.
- The recombination process is predominantly given through the reactions

$$p + e^- \rightarrow H + \gamma, \quad (25)$$

$$H + \gamma \rightarrow p + e^-. \quad (26)$$

Origin of the CMB (5)

- In the case of statistical equilibrium, the number density of different chemical species i is given via the Maxwell-Boltzmann distribution:

$$n_i = g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{(\mu_i - m_i)/T}, \quad (27)$$

with g_i the number of internal degrees of freedom, m_i the mass, μ_i the chemical potential and T the temperature.

- Chemical reactions minimize the net chemical potential $\mu = \sum \mu_i$.
- Evaluating Eq. (27) for protons (p), electrons (e) and atomic hydrogen (H), one can show that

$$\frac{n_p n_e}{n_H} \sim e^{-B/T} \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{(\mu_p + \mu_e - \mu_H)/T}, \quad (28)$$

with $B = m_p + m_e - m_H = 13.6$ eV the binding energy of atomic hydrogen, and $g_p = g_e = \frac{1}{2} g_H = 2$.

Origin of the CMB (6)

- In a state of chemical equilibrium, we have $\mu_p + \mu_e = \mu_H$, leading to the simplified relation

$$\frac{n_p n_e}{n_H} \sim e^{-B/T} \left(\frac{m_e T}{2\pi} \right)^{3/2} . \qquad (29)$$

- We now define the ionized fraction x_e of hydrogen as

$$n_p = n_e = x_e n_B , \qquad (30)$$

$$n_H = n_b - n_p = (1 - x_e) n_B , \qquad (31)$$

with n_B the number density of the baryons.

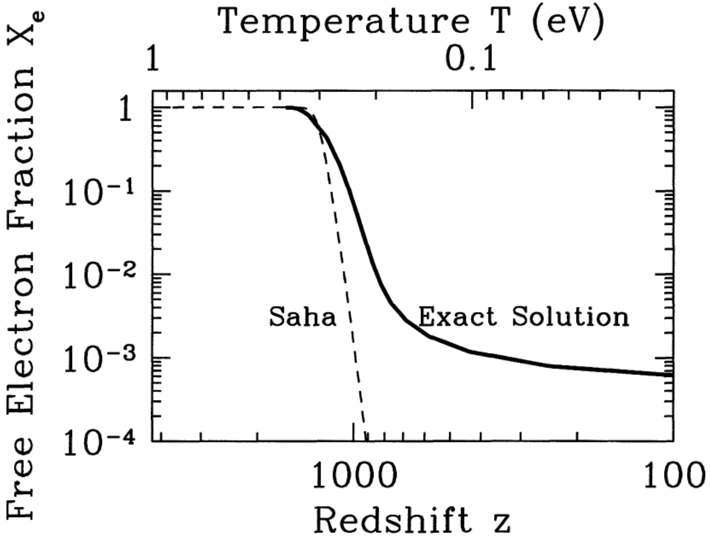
- We can then rewrite Eq. (29) as the Saha equation:

$$\frac{n_e n_p}{n_H n_B} = \frac{x_e^2}{1 - x_e} = \frac{1}{n_B} \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-B/T} . \qquad (32)$$

Origin of the CMB (7)

- From the exponential term of the Saha equation, one could expect that recombination occurs when $T \sim B$, i.e. when the temperature of the Universe is about equal to the binding energy of atomic hydrogen.
- With $B = 13.6$ eV, the latter would imply a temperature $T \sim 1.6 \times 10^5$ K.
- With the relation $T_{\mathrm{CMB}} = T_{\mathrm{CMB},0}(1+z)$, the latter would correspond to a redshift of $z \sim 5.8 \times 10^4$.
- However, evaluating the Saha equation, one actually finds that recombination happens much later, more closely to $z \sim 1000$!
- The latter can be shown to be related to the high photon-to-baryon ratio of $\sim 10^9$, due to high-frequency photons keeping the Universe ionized.

Origin of the CMB (8)



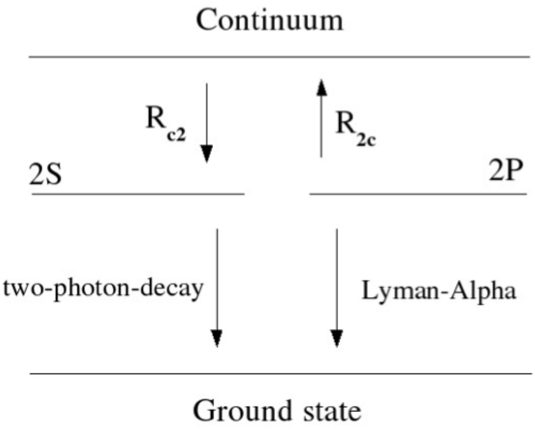
Origin of the CMB (9)

- In the general case, one has to solve an equation solving the non-equilibrium evolution both for hydrogen and helium.
- We denote here the ionized fraction of hydrogen/helium as x_i, with $i =$H,He. We then have:

$$\frac{dx_i}{dt} = \alpha_B n_{Hp} x_e x_i, \quad (33)$$

with n_{Hp} the total number density of hydrogen plus protons, x_e the total ionization fraction ($x_e = x_i$) for a pure hydrogen gas.

Origin of the CMB (10)



Recombination of atomic hydrogen.

Origin of the CMB (11)

- Direct recombinations to the ground state will release energetic photons, which will directly ionize a neighboring atom (no net effect).
- A recombination to the excited state of atomic hydrogen however yields a photon cascade.
- In particular the 2s-1s transition procedes via the emission of two photons and allows no subsequent ionization.
- A detailed modeling of these processes yields the evolution of the ionization degree as

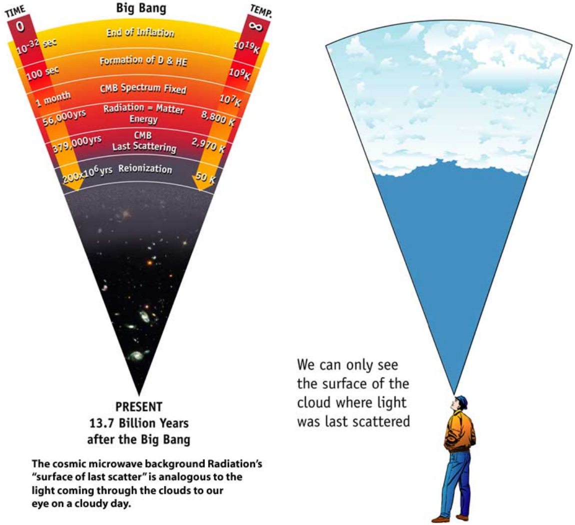
$$x(z) = 2.4 \times 10^{-3} \frac{\sqrt{\Omega_m h^2}}{\Omega_b h^2} \left(\frac{z}{1000} \right)^{12.75} \tag{34}$$

for $800 < z < 1200$.

Origin of the CMB (12)

- As a result of the rapid recombination, the Thompson scattering optical depth decreases substantially with redshift.
- The radiation in the Universe can thus propagate without any further interaction.
- The epoch of recombination is thus also referred to as the **epoch of last scattering**.
- The density structure from that epoch is thus imprinted in the CMB radiation we observe today!

Origin of the CMB (13)



Recombination as the last scattering surface

Origin of the CMB (14)

- In general, the recombination rate in the Universe is given as $k_{\rm rec} n_e n_p$, with $k_{\rm rec}$ the temperature-dependent recombination coefficient.
- The recombination timescale is thus

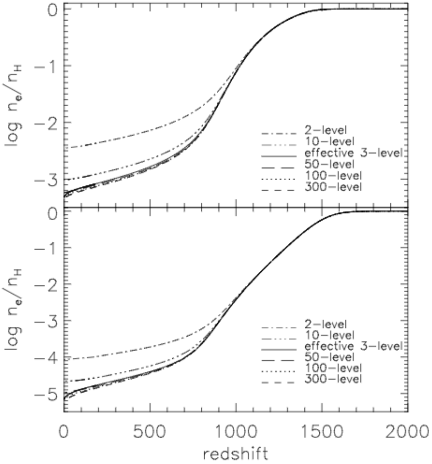
$$\tau_{\rm rec} = \frac{n_e}{k_{\rm rec} n_e n_p} = \frac{1}{k_{\rm rec} n_p} = \frac{1}{k_{\rm rec} x_e n_B} \propto x_e^{-1} a^3. \quad (35)$$

- The time available for recombinations is roughly the age of the Universe at redshift z, i.e.

$$t \propto a^{3/2} = \left(\frac{1}{1+z}\right)^{3/2}. \quad (36)$$

- The timescale required for recombinations thus increases more rapidly than the age of the Universe.
- Recombination will thus become highly inefficient, leading to a constant ionization degree with $x_e \sim 2 \times 10^{-4}$ (freeze-out).

Origin of the CMB (15)



Treatment of hydrogen as a multi-level atom for two cosmological models (top: standard model, bottom: $\Omega_B = \Omega_{\rm tot} = 1$). Seager et al. (2000).

Origin of the CMB (16)

- Publicly available: RECFAST code
  http://www.astro.ubc.ca/people/scott/recfast.html
- Solves hydrogen and helium recombination for different cosmological models, reproducing results of detailed multi-level calculations.
- Available with Fortran and C++.
- Documentation: Seager, Sasselov & Scott (1999).